MECHANICS.

198. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Three spheres of the same material, radii R, r, S, rest upon a horizontal plane, touching each other. Find the radius of a sphere of the same material as the others which, being placed upon the other three spheres, will just prevent the latter from separating, the coefficient of friction between the spheres being μ, and between the spheres and the table being μ'

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let x=radius of top sphere center Q, R=radius of bottom sphere center P. The forces acting on the sphere radius R are its own weight W, the friction F, the friction F', and the normal reaction between the two spheres R. Let nW_1 be the part of the weight of the sphere radius x supported by the sphere radius R. $\angle DPC=\beta$.

Now $FHC=RDC$. $F=R\left(\dfrac{DC}{HC}\right)=R\tan\frac{1}{2}\beta$. $\therefore F=\mu R$ or $\mu=\tan\frac{1}{2}\beta$.

$F'=\mu'[W+nW_1]=R\sin\beta-F\cos\beta=R(\sin\beta-\tan\frac{1}{2}\beta\cos\beta]=R\tan\frac{1}{2}\beta$.

$$\therefore R=\frac{\mu'[W+nW_1]}{\tan\frac{1}{2}\beta}=\frac{\mu'}{\mu}[W+nW_1].$$

Resolving vertically, $nW_1=R\cos\beta+F\sin\beta$.

$$\therefore nW_1=R[\cos\beta+\tan\tfrac{1}{2}\beta\sin\beta]=R. \quad \therefore nW_1=\frac{\mu'}{\mu}[W+nW_1].$$

Let δ=density of each sphere. $\therefore W=\delta g\times\frac{4}{3}\pi R^3$, $W_1=\delta g\times\frac{4}{3}\pi x^3$.

$$\therefore nx^3=\frac{\mu'}{\mu}[R^3+nx^3]. \quad \therefore n=\frac{\mu' R^3}{[\mu-\mu']x^3}.$$

Let mW_1 be the part of the upper sphere supported by sphere radius r, and pW_1 the part supported by sphere radius S. Then

$$m=\frac{\mu' r^3}{[\mu-\mu']x^3},\quad p=\frac{\mu' S^3}{[\mu-\mu']x^3}.$$

But $n+m+p=1$.

$$\therefore \frac{\mu'[R^3+r^3+S^3]}{\mu-\mu'}=x^3,\text{ and } \therefore x=\sqrt[3]{\frac{\mu'[R^3+r^3+S^3]}{\mu-\mu'}}.$$

199. Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

A sphere of water, radius $\frac{1}{49}$, the earth's radius, is brought together by mutual attractions of particles from a state of infinite diffusion. Find its temperature owing to the amount of work done by these forces.

Solution by W. D. LAMBERT, Washington, D. C.

Let r be the variable radius of the sphere, while forming; a the final radius $=\dfrac{6.370\times10^8}{49}$ centimeters; ρ the density of water$=1$; k the gravitation constant$=6.665\times10^{-8}$ dynes; J the mechanical equivalent of heat$=4.184\times10^7$ ergs for the centigrade gram-calorie. It is a little easier to conceive the sphere as pulled asunder against its own attraction, and the amount of work will be the same. Suppose the sphere made up of layers, each of thickness dr, and that the sphere has been reduced to radius r. The mass of a layer is $4\pi\rho r^2dr$. The attraction between this mass and the remaining sphere is $\dfrac{k\times\frac{4}{3}\pi\rho r^3\times4\pi\rho r^2dr}{x^2}$, where x denotes the distance of the layer (supposed to be scattered symmetrically) from the center of the sphere. The work done in removing the layer from the surface of the sphere in question to infinity is $\frac{16}{3}\pi^2k\rho^2r^5dr\int_r^\infty\dfrac{dx}{x^2}=\frac{16}{3}\pi^2k\rho^2r^4$.

The total work done in removing all layers is

$$\tfrac{16}{3}\pi^2k\rho^2\int_0^a r^4dr=\tfrac{16}{15}\pi^2k\rho^2a^5.$$

Dividing this quantity by the mass and by J we get for the temperature $\frac{4}{5}\dfrac{\pi k\rho a^2}{J}$. For substances other than water this result should be multiplied by the specific heat of the substance. Using the numerical values previously given, we get for the temperature $0^\circ.677$ centigrade.

Also solved by G. B. M. Zerr, whose result is 0.656. This difference of result is due to the different values assumed for the constants entering into the solution.

AVERAGE AND PROBABILITY.

183. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

A point within a given triangle is joined to each of the corners. What is the average of the sum of the lengths of these three lines?

I. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let ABC be the given triangle, P the random point, A the vertex, BC the base of the triangle, AD the altitude. Through P draw QR, parallel to BC cutting AD in F. Let $AD=p$, $BD=e$, $DC=d$, $AF=x$, $FP=y$. Then $AP=\sqrt{[x^2+y^2]}$. The limits of x are 0 and p; of y, $-QF=ex/p$ and $+FR=dx/p$. Let M=average length of AP, Δ=average length of the sum.

$$\therefore M=\int_0^p\int_{-ex/p}^{dx/p}\sqrt{[x^2+y^2]}\,dxdy\Big/\int_0^p\int_{-ex/p}^{dx/p}dxdy$$